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Flyby Error Analysis Based on Contour Plots for Cassini Tour

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The maneuver cancellation analysis consists of cost contour plots employed by the Cassini maneuver team. The plots are two-dimensional linear representations of a larger six-dimensional solution to a multimaneuver, multiencounter mission at Saturn. This realization and the use of the tool itself is just one of the many significant engineering achievements that have come from the Cassini project. After inserting the cost contour capability with an enhancement (taking account of asymptote changes), a tool that was originally only used for analysis could be used for operations once the accuracy of the plots was determined to be acceptable for operations. The plots have been used extensively since the enhancement. By using contours plotted in the B plane with $B \bullet R$ and $B \bullet T$ components, it is possible to view the effects on ΔV for various encounter positions in the B plane. The plot is used in operations to help determine if the approach maneuver (ensuing encounter minus three days) and/or the cleanup maneuver (ensuing encounter plus three days) can be cancelled and is a linear check of an integrated solution. The plots have also been used to bias the targets of encounters to save ΔV.

Nomenclature

B	=	B vector or B plane
$B \bullet R$	=	vertical axis of the B plane
$B \bullet T$	=	horizontal axis of the B plane
$C3$	=	excess velocity or characteristic energy
$G_{x,y}$	=	grid point value on the contour plot for an approach maneuver, m/s
$G_{x,y,c}$	=	grid point value on the contour plot for a cleanup maneuver, m/s
i	=	leg index
$J_{x,y}$	=	ΔV calculation of a B plane grid point for an approach maneuver, m/s
$J_{x,y,c}$	=	ΔV calculation of a B plane grid point for a cleanup maneuver, m/s
K	=	K matrix, a maneuver capability matrix
M	=	the B plane mapping
R	=	R vector of the B plane
r_o	=	nominal trajectory apoapsis maneuver location
S	=	the incoming asymptote
T	=	T vector of the B plane
T_n	=	nth encounter of Titan
$V1$	=	the magnitude of the velocity for a cleanup maneuver, m/s
$V2$	=	the velocity magnitude for an apoapsis maneuver, m/s
ΔB_A	=	asymptote change B plane state vector
ΔB_{RT}	=	grid variation vector
ΔB_{xy}	=	the variations in the B plane with a perturbed trajectory
$\Delta B \bullet R_y$	=	y component of grid points in the B plane
$\Delta B \bullet T_x$	=	x component of grid points in the B plane
Δr	=	difference in the apoapsis maneuver location for a perturbed trajectory
ΔV	=	change in velocity, m/s
ΔV_{REF}	=	change in velocity given in the reference trajectory, m/s

I. Introduction

THE Cassini–Huygens mission to Saturn launched on 15 October 1997 and successfully entered Saturn's orbit on 1 July 2004. The orbital phase started after Saturn orbit insertion and continued through the prime mission, which ended formally on 30 June 2008 with the extended mission commencing after this date. The primary methods for changing orbital parameters during the Cassini–Huygens orbital segment are the gravity-assist encounters with Titan, Saturn's largest moon. In between encounters with Titan or any other Saturnian moon, there are typically three opportunities for orbit trim maneuvers (OTMs). These three maneuvers, shown in Fig. 1, are named the cleanup maneuver (which takes place approximately three days after an encounter), the apoapsis maneuver or shaping maneuver (which usually takes place around apoapsis of the orbit), and the approach maneuver (which takes place approximately three days before the next encounter). The targets for the maneuvers are $B \bullet R$ and $B \bullet T$ components of the B plane and linearized flight time (LFT) [1]. A description of the B plane is in the Appendix.

During the 11 years since launch, there have been many trajectory correction maneuvers during interplanetary cruise, OTMs implemented throughout the orbital phase, and a number of canceled maneuvers [2]. Preserving the science objectives is one of the navigation team's goals, and so the decision to cancel a maneuver, implement a maneuver, or bias an encounter target has to take account science implications. In order for the Cassini–Huygens maneuver team to provide information for project management to determine if cancellation of a maneuver was the appropriate course of action, tools were developed that would quantify what would happen with and without a particular maneuver. Thus, the maneuver cancellation analysis software was developed [3]. The maneuver cancellation analysis software runs every time the maneuver team designs a maneuver solution for the ensuing encounter. One product of the maneuver cancellation analysis software is the ΔV cost contour plot.

The contour plot is a two-dimensional linear representation of a larger six-dimensional solution to the multimaneuver, multiencounter operation that takes place on the Cassini–Huygens mission at Saturn. The plots show ΔV costs for different B plane encounter

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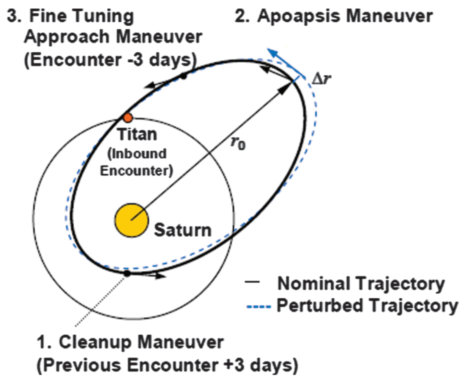


Fig. 1 Maneuver locations of a typical Cassini–Huygens encounter with Titan along with the illustration of a nominal and perturbed trajectory.

positions of the spacecraft with respect to the nominal B plane encounter aimpoint. There are two versions of the plot: one for the approach maneuver and another for the cleanup maneuver. The two types of contour plots are generated by using the linear analysis of maneuvers with bounds and inequality constraints (LAMBIC) computer program, which produces the statistics of ΔV magnitude and delivery accuracy by simulating the execution of a sequence of maneuvers through the use of the Monte Carlo method. Included in LAMBIC is an optimization routine that maneuver analysts use to develop maneuver optimization strategies and design OTMs. LAMBIC [4] can also generate ΔV costs corresponding to B plane grid points that represent flyby errors. Data associated with the grid points are used to make the contours, but the LAMBIC program does not make the plots. The contour plots are produced by MATLAB [5] with the grid point data from LAMBIC.

The purpose of the cost contour plot was to add another tool for the maneuver analysts to help determine, along with management, whether to cancel a maneuver. The contour plot has become an important part of our analysis and will become increasingly important as the Cassini–Huygens mission continues. It is a significant tool that was used on the Galileo project but has since been enhanced and updated for Cassini–Huygens. It was determined, after multiple comparison matches between the contour plots and integrated solutions of the maneuver cancellation ΔV cost, that the contour plots are usable for operations. Consequently, the enhancement modified the analysis tool LAMBIC into a tool also used for operations.

II. Methodology and Development

The contour plots are made by plotting ΔV costs for a grid of B plane data points, which shows changes in ΔV for various encounter positions in the B plane, as shown in Fig. 2. Plot a of Fig. 2 is the contour plot whereas plot b is the contour surface. Plotting the ΔV contours on the B plane allows for the maneuver implementation delivery ellipse and current orbit determination (OD) delivery ellipse to be drawn on the plot. (See Appendix for B plane information.) Subsequent application of statistics to the contour plot gives the navigation team probabilities that certain ΔV costs will or will not occur.

Originally, the approach maneuver contour plot was used to show the sensitivity of flyby errors. Such items include determining if the nominal flyby aimpoint represents a classical quadratic minimum and if the encounter would be on the flyby altitude boundary (i.e., the minimum altitude required to prevent the spacecraft from tumbling due to atmospheric drag), all providing limited value. However, after some appropriate modifications to take account for the change between incoming nominal and perturbed asymptotes, the approach maneuver cost contour plot has become an important tool in the decision-making process of canceling approach maneuvers. The modifications will be further covered in Sec. III.

It was then theorized that a similar contour plot could be constructed for the cleanup maneuvers. The cleanup maneuver cost contour is formed by differencing two LAMBIC runs and then mapping that difference back to the encounter just before the cleanup maneuver. The first LAMBIC run is with the cleanup maneuver included in the simulation while the second LAMBIC run is without the cleanup maneuver. The reason for mapping the cost contours to the previous flyby is that there is not a well-defined B plane target at the next encounter for the cleanup maneuver. This is because the cleanup maneuver and the apoapsis maneuver (the maneuver between the cleanup and approach maneuvers) are designed together and the intermediate target is flexible. (The target of the cleanup maneuver is the intermediate target. The intermediate target is shown as $r_o + \Delta r$ in Fig. 1.)

While using the contour plots, another application for these tools developed. The approach maneuver contour plot became useful for biasing the targets of the approach maneuvers. As a result, there have been two instances where the B plane targets of an encounter were changed because a sizable negative ΔV region (a negative ΔV region represents an area of ΔV savings) was discovered on the approach maneuver contour plot that was big enough to target. Therefore, the encounter target changed to take advantage of the ΔV savings. This will be further discussed in Sec. IV. In addition, the contour plots help

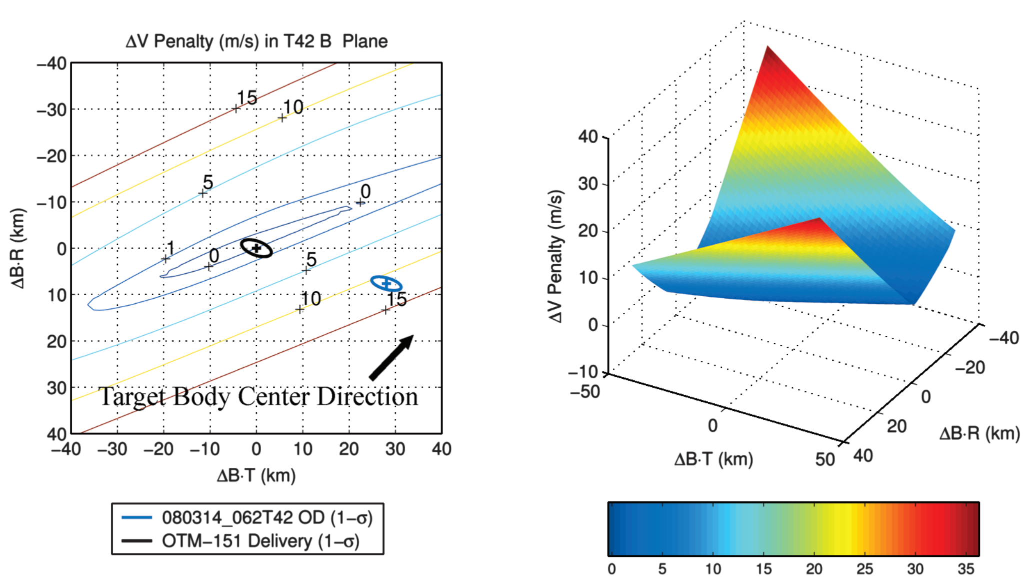


Fig. 2 Graphs showing a) Titan encounter 42 approach maneuver contour plot, and b) Titan encounter 42 approach maneuver contour surface.

the maneuver analysts determine that the current setup for a particular maneuver is correct, and, more importantly, it is a linear check of the nominal integrated solution. The contour plots also take into account future encounters and how the current encounter target effects the downstream ΔV costs associated with those future encounters. It can also be seen how the future encounters affect the ΔV cost at the next encounter. This observation is evident in the changes to the contour plots when the length of the optimization chain is changed for various LAMBIC runs.

The contour lines are shaped and oriented in a manner that corresponds to the orbital change that takes place due to the flyby of the encounter body. For instance, if the altitude of the flyby is the primary cause of the orbital change, the semiminor axis of the contours will line up with the B vector of the encounter. Alternatively, stated another way, if the orbital change that takes place because the encounter is sensitive to the altitude of the encounter, the steepest gradient of the contour lines will line up with the B vector.

On the approach maneuver contour plot in Fig. 2 are two one-sigma ellipses. The blue ellipse is the current trajectory without a maneuver taking place, which the orbit determination team provided to the maneuver team. The nomenclature shown in the legend for the blue ellipse represents a specific OD delivery. The first six numbers represent the date of the delivery in the YYMMDD format. For example, the legend for Fig. 2a is for the OD delivery on 080314 or March 14, 2008. The other values represent the revolution number and encounter identifier. The Fig. 2a OD ellipse name indicates the 62 Saturn revolution and the Titan 42 encounter. The black ellipse is the delivery ellipse if the maneuver takes place. Figure 2 shows in the legend that this contour plot is for OTM151 analysis. Included in the delivery ellipse are the one-sigma maneuver execution errors [6]. The axes of the contour plot are position differences in $B \bullet R$ and $B \bullet T$ with respect to the nominal aimpoint. Therefore, the nominal aimpoint is at point (0,0). The contour values are ΔV differences with respect to the ΔV value for the nominal aimpoint. This is why the zero contour always passes through the origin of the plot (More in-depth information on this topic is in Sec. III). The reason why ΔV differences with respect to the nominal aimpoint are used is that the ΔV difference is more consistently accurate than the absolute ΔV values.

III. Theory

A. Approach Maneuver Cancellation Contour Plot

The ΔV costs shown on the contour come from two sources: B plane grid variations (encounter position variations) and an asymptote correction for offsets between distinct B planes for the nominal and perturbed trajectory. Figure 3 shows an example of the B plane used for the contour plot with a sample grid point.

Each grid point is a summation of the ΔV for multiple maneuvers over multiple encounters. For the approach maneuver case, the algorithm that the optimizer uses is the following cost function:

$$J_{x,y} = \sum_{i=0}^{3} (\|V1_i\| + \|V2_i\|) \tag{1}$$

$J_{x,y}$ represents the ΔV calculation for a grid point, $V1$ represents the magnitude of the velocity for a cleanup maneuver, and $V2$ represents the velocity magnitude for an apoapsis maneuver. The i index represents a leg number. Usually four legs or four sets of cleanup and apoapsis maneuvers are used in the optimization process. Now the value at each grid point is subtracted from the cost at the nominal aimpoint. (The nominal aimpoint is at the origin of the contour plot.) So the value of the grid point is computed from:

$$G_{x,y} = J_{x,y} - J_{0,0} \tag{2}$$

$G_{x,y}$ represents the values on the contour plot. Note that this is the cost due to flyby error assuming the ensuing cleanup maneuver will be implemented, the approach maneuver is assumed to be cancelled, and the encounter takes place at the grid points. The grid variation can be written in the following equation because the only terms that have a value are the $B \bullet R$ and $B \bullet T$ terms:

$$\Delta B_{RT} = \begin{bmatrix} \Delta B \bullet R_y \\ \Delta B \bullet T_x \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3}$$

Figure 4 is an illustration of the B plane offset that goes with Fig. 1, which is an example of a perturbed trajectory (perturbed with respect to the nominal reference trajectory). $\boldsymbol{B_o}$ is the B vector for the nominal trajectory, $\boldsymbol{S_o}$ is the incoming asymptote for the nominal trajectory, T_n is the nth Titan encounter, T_{n+1} is the $n+1$th Titan encounter, $\boldsymbol{B}$ is the B vector for the perturbed trajectory, and $\boldsymbol{S}$ is the incoming asymptote for the perturbed trajectory.

Figure 4 illustrates the offset in the B plane due to the incoming asymptote being different from the asymptote of the original trajectory. The target at T_n is the same but the "route" to get to the target changed from the one that was planned. This asymptote change can be written in B plane state as

$$\Delta B_A = \begin{bmatrix} \vec{B} \bullet \vec{R}_o \\ \vec{B} \bullet \vec{T}_o \\ \mathrm{LFT} \\ \vec{S} \bullet \vec{R}_o \\ \vec{S} \bullet \vec{T}_o \\ C3 \end{bmatrix} - \begin{bmatrix} \vec{B}_o \bullet \vec{R}_o \\ \vec{B}_o \bullet \vec{T}_o \\ \mathrm{LFT}_o \\ \vec{S}_o \bullet \vec{R}_o \\ \vec{S}_o \bullet \vec{T}_o \\ C3_o \end{bmatrix} \tag{4}$$

The LFT term has negligible effect. Therefore, it is set to zero. Also, $\vec{S}_o \bullet \vec{R}_o$ and $\vec{S}_o \bullet \vec{T}_o$ terms are zero by definition. Equation (5) brings together ΔB_A and ΔB_{RT}

$$\Delta B_{x,y} = \Delta B_A + \Delta B_{RT} \tag{5}$$

$$\Delta B_{x,y} = \begin{bmatrix} (\vec{B} - \vec{B}_o) \bullet \vec{R}_o \\ (\vec{B} - \vec{B}_o) \bullet \vec{T}_o \\ \approx 0 \\ \vec{S} \bullet \vec{R}_o \\ \vec{S} \bullet \vec{T}_o \\ \Delta C3 \end{bmatrix} + \begin{bmatrix} \Delta B \bullet R_y \\ \Delta B \bullet T_x \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

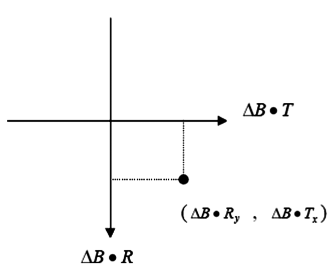


Fig. 3 Example of a *B* plane with grid point sample.

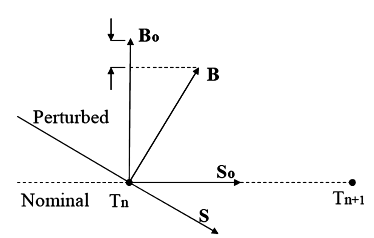


Fig. 4 Illustration of a *B* plane offset, which is due to the difference of the perturbed trajectory incoming asymptote and the nominal trajectory incoming asymptote.

$\Delta B_{x,y}$ represents the variations in the B plane (due to the ΔB_{RT} term) using the perturbed trajectory B plane state. $\vec{B}$ is the perturbed B vector, $\vec{B}_o$ is the nominal B vector, $\vec{R}_o$ is the nominal B plane R vector, $\vec{T}_o$ is the nominal B plane T vector, $\vec{S}$ is the perturbed incoming asymptote, and $\Delta C3$ is the $C3$ difference. The $\Delta B \bullet T$ component is the x grid point of the B plane, with the $\Delta B \bullet R$ component the y grid point. So the B plane variation mapped (M is the B plane state mapping) from the nth Titan encounter to the $n+1$th Titan encounter for the perturbed trajectory is

$$M_n^{n+1} * \Delta B_{x,y} \tag{7}$$

A special setting in LAMBIC produces the ΔB_{RT} component. This special setting produces ΔV values that correspond to the grid of position errors with respect to the flyby. When merged with another feature of LAMBIC, the aforementioned special setting has the function of centering the grid of flyby errors at the perturbed flyby conditions represented by the incoming asymptote error estimate. The asymptote error estimate is computed by using the current trajectory from operations. Thus, the result is a grid of ΔV values in the B plane that corresponds to the perturbed trajectory, which is much more accurate than using the nominal or reference trajectory B plane.

The influence of using the current trajectory from operations rather than the nominal reference trajectory can be seen in the difference between Figs. 5 and 6, which are contour plots of the T32 encounter. Figure 5 shows the case for approach maneuver OTM115 without applying the incoming asymptote correction. It shows that the current OD ellipse is close to the 5 m/s contour, the delivery ellipse is directly on top of the zero m/s contour point, and there is no negative ΔV region. Hence, the best maneuver strategy would be targeting to the nominal aimpoint. Figure 6 shows the cost contour for the same maneuver, but this plot takes into account the incoming asymptote change. The OD ellipse is not as close to the 5 m/s contour as in Fig. 5, the delivery ellipse is still on the zero m/s contour but the zero m/s contour is now an oval rather than a point, and there is a substantial negative ΔV region. Now, targeting to the nominal aimpoint does not look like the best option. (Note: The project decided to bias the encounter targets for the T32 encounter. There is more discussion on biased targeting in Sec. IV.) This proves how important the asymptote correction is in accurately displaying the ΔV costs associated with a flyby.

B. Cleanup Maneuver Cancellation Contour Plot

During operations, the possibility of cancelling a cleanup maneuver emerged. A cleanup maneuver contour plot was formulated to consider cancellation of a cleanup maneuver. There are differences to the production of the cleanup maneuver contour plot compared with the approach maneuver contour plot.

For the cleanup maneuver cancellation case, the assumption is that the cleanup maneuver is cancelled. The subsequent apoapsis maneuver is implemented. As mentioned before there are two LAMBIC runs performed that generate the data for the cleanup maneuver contour plot. The difference between the two LAMBIC runs produces the ΔV cost values of the contour plot. The first run is the nominal run that produces values from Eq. (1). The second run uses a different cost function that is the minimization of

$$J_{x,y,c} = \|V2_0\| + \sum_{i=1}^{3}(\|V1_i\| + \|V2_i\|) \tag{8}$$

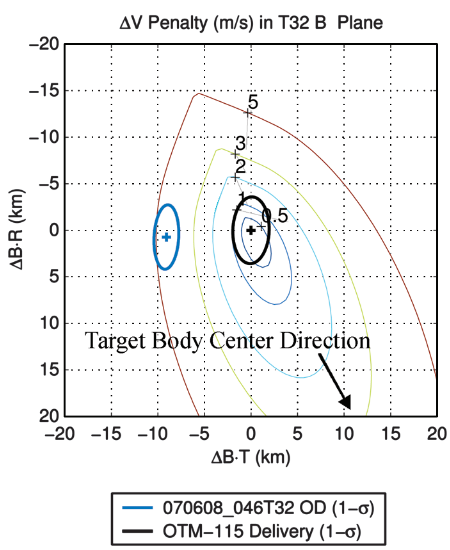
Where $J_{x,y,c}$ represents the ΔV calculation for a grid point, $V2_0$ represents the upcoming apoapsis maneuver ($V1_o$ is cancelled). Usually, four legs or four sets of cleanup and apoapsis maneuvers are used in the optimization process, but for this case, there are only three legs in the summation. Let $G_{x,y,c}$ be the difference of the ΔV at a grid point without the cleanup maneuver minus the ΔV at a grid point with the cleanup maneuver

$$G_{x,y,c} = J_{x,y,c} - J_{x,y} \tag{9}$$

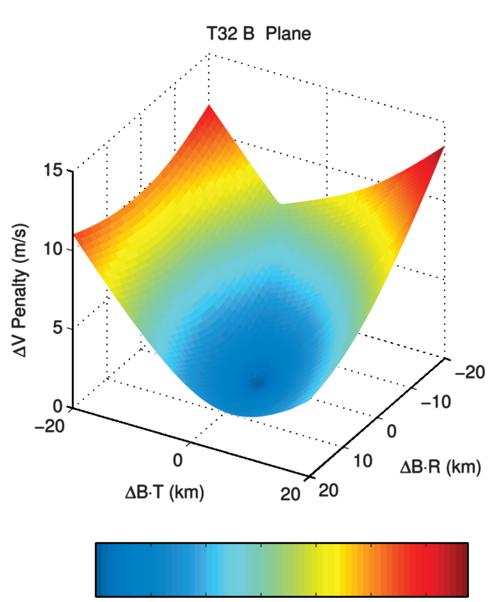
The B plane error mapped from the nth Titan encounter to the $n+1$th Titan encounter for the perturbed trajectory, assuming cancellation of the cleanup maneuver, is

$$M_n^{n+1} * \Delta B_{xy} - K_{6\times3}\Delta V_{\mathrm{REF}} \tag{10}$$

The variations ΔB_{xy} for the cost contours are with respect to the previous flyby, not the ensuing flyby, and M is the B plane mapping. The $-K_{6\times3}\Delta V_{\mathrm{REF}}$ component of Eq. (10) is included in order to subtract or take out the cleanup maneuver, which is nominally included with the reference trajectory because it might have a deterministic component. (If a cleanup maneuver with a deterministic component is cancelled, its effect needs to be accounted for in the LAMBIC simulation. The subtraction is done by adding the negative influence of the maneuver.) The K term is the maneuver capability matrix at the cleanup maneuver, and the ΔV_{REF} is the nominal deterministic ΔV planned in the reference trajectory. Multiplying

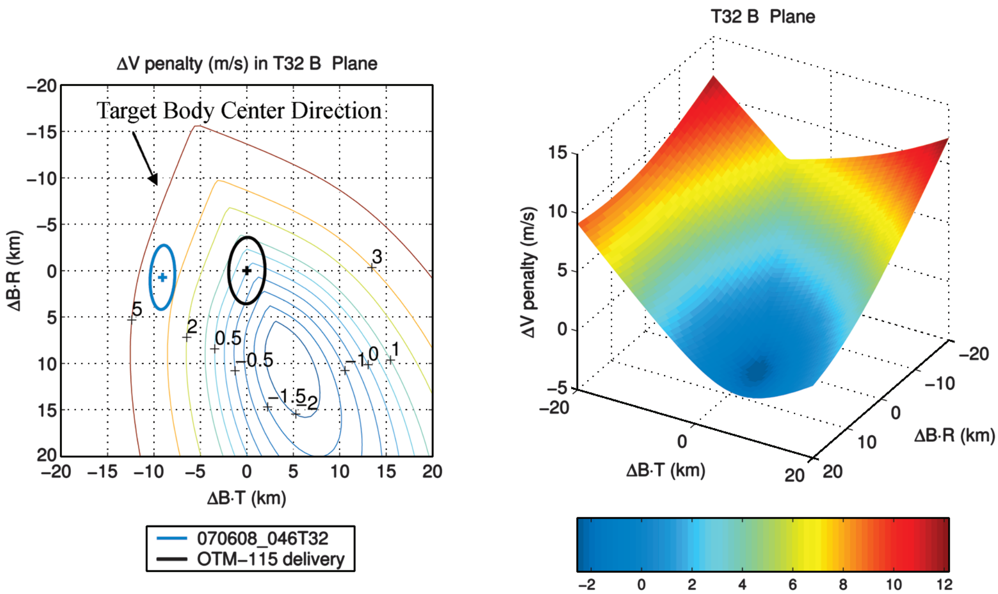


a)



b)

Fig. 5 T32 approach maneuver OTM115 cancellation cost without incoming asymptote correction.



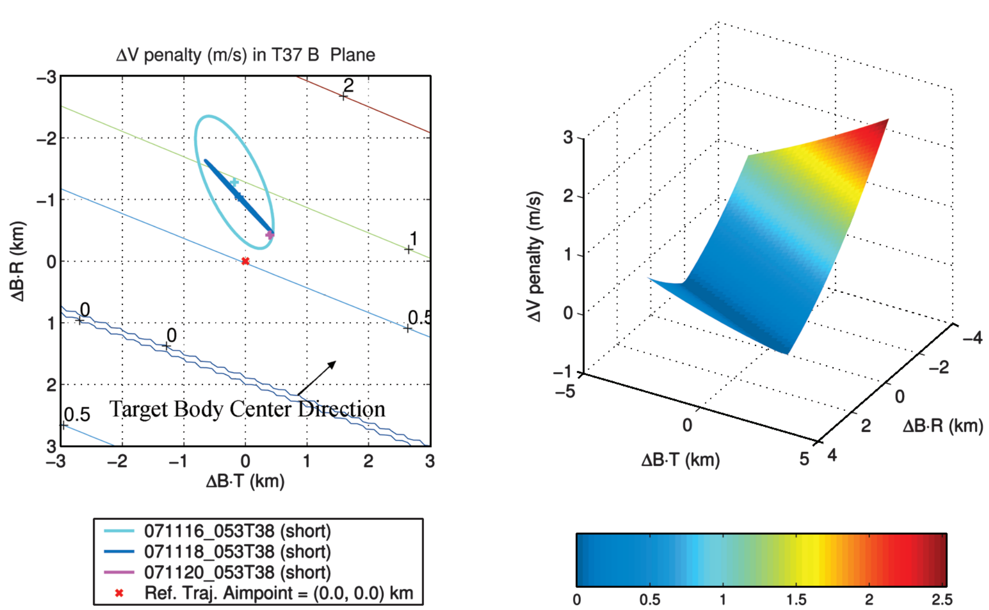
a) b)

Fig. 6 Contour plot for OTM115 with the incoming asymptote correction showing the negative ΔV contours before the target bias.

these together gives the B plane change due to the nominal (reference) cleanup maneuver.

Figure 7 is an example of a cleanup maneuver contour plot where there was a deterministic ΔV component of 1.17 m/s. The plot shows the contours running parallel with a zero contour channel. The (0,0) point of the contour is the nominal trajectory aimpoint, but the zero m/s cost contour usually does not pass through it. On the contour, three OD solutions are plotted. Two OD solutions, 071116_053T38 (short) and 071118_053T38 (short), are pre-encounter solutions mapped to the previous B plane. The magenta "+" is the last OD solution, 071120_053T38 (short). It is a posten-counter solution, which shows the actual encounter position in the B plane. For this cleanup maneuver contour plot example, using the 071120_053T38 (short) solution shows that the cost to cancel OTM134 [7] is approximately 0.7 m/s. This was in very close agreement with the integrated solution, which produced a value of approximately 0.8 m/s OTM134 cancellation cost. Notice that there would be a 0.5 m/s cost to cancel OTM134 even with the achieved nominal trajectory aimpoint.

Figure 8 shows how to subtract the cost functions used for both the cleanup maneuver contour plot and the approach maneuver contour plot in order to create the grid point values. Using the matrix, the cleanup maneuver contour plot is made by calculating the grid points $G_{x,y,c}$ by subtracting the terms in the matrix horizontally; explicitly, subtracting the $J_{x,y,c}$ term (the cleanup maneuver cancellation cost function) by the $J_{x,y}$ term (the cleanup maneuver implementation cost function). An example of this subtraction for point $x = 1$ and $y = 2$ would be $J_{1,2,c}$ subtracted by $J_{1,2}$ producing $G_{1,2,c}$ [Eq. (9)]. Another example of this subtraction is shown in Fig. 8 where $J_{0,0,c}$ subtracted by $J_{0,0}$ generate $G_{0,0,c}$. $G_{0,0,c}$ is the grid point value at the origin for the cleanup maneuver contour plot. In the case for the approach maneuver contour plots, the grid points for the contour plot are made by subtracting the terms vertically. Specifically, subtract the $J_{x,y}$ cost function value terms at the various



a) b)

Fig. 7 Graphs showing a) cleanup maneuver cancellation cost contour plot for OTM134, and b) cleanup maneuver cancellation cost surface plot for OTM134.

		Cleanup Maneuver Contour Plot	
		Cancel	Implement
Approach Maneuver Contour Plot	Cancel	$J_{x,y,c}$	$J_{x,y}$
	Implement	$J_{0,0,c}$	$J_{0,0}$

Fig. 8 Contour plot grid value cost function matrix.

grid points (which represents cancellation of the approach maneuver or the cost of a nonexact encounter taking place) by the $J_{0,0}$ cost function value (which represents the implementation of an approach maneuver) as shown in Eq. (2). Figure 8 shows another calculation that is not used on the Cassini maneuver team. Using the matrix, it is a vertical subtraction of terms, particularly subtracting $J_{0,0,c}$ from $J_{x,y,c}$. This subtraction would construct the cost contour points for canceling both the approach maneuver and the cleanup maneuver simultaneously.

IV. Biasing the Encounter Target

As mentioned previously, the contour plots are used to estimate the cost of maneuver cancellation, and the one-sigma OD and maneuver delivery ellipses can provide some statistics on the cost of cancellation. One more use should be discussed in some detail because it saved ΔV for the Cassini–Huygens Project. This use is biasing the targets of an encounter. Biasing or changing targets is not new. The Galileo project changed targets; however, that mission had more time to adjust the targets. For Cassini–Huygens the time to design a maneuver and make all the appropriate decisions is about three days (given a typical five-day spacing of OTMs). Therefore, the quick time frames do not usually permit retargeting. However, there were some instances where the contour plot aided in target biasing because it was simple to take the appropriate information from the plot. Biasing the encounter targets was done twice (for OTM109 and OTM115, which targeted to T30 and T32, respectively) and saved approximately 4.6 m/s of ΔV. The original contour plot for OTM115 is shown in Fig. 6. In Fig. 6, there is a sizable negative ΔV area, as mentioned in Sec. III. It was determined that changing the targets did not have any undesirable downstream effects, and the trajectory would be closer to the reference trajectory by targeting the maneuvers to biased aimpoints. Changing the $B \bullet R$ target by 9.4 km and the $B \bullet T$ target by 4.1 km without changing the encounter time would put the center of the delivery ellipse at the absolute minimum of the contour plot. Figure 9 is the contour plot after the B plane target changed to the biased target. The decision to bias the target was not just a navigation and ΔV savings issue. There had to be discussion with the science team to make sure that the biased target did not negatively affect planned science activities. However, because there were two benefits (the 4.6 m/s of ΔV savings and the trajectory would be closer to the reference trajectory by implementing the maneuver with the biased targets), the science team agreed to the change.

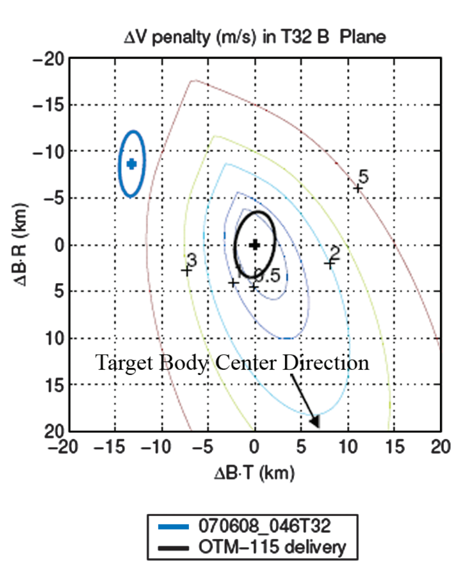
V. Future Work

The contour plots show very accurate results when dealing with a Titan encounter. However, there is not the same level of accuracy for a contour plot of the smaller and less massive Saturnian satellites such as Enceladus. It is hypothesized that the reason for the inaccuracy is due to the asymptote difference being the dominant variable for ΔV costs in the less massive satellite encounters whereas for a Titan encounter the grid points, or more specifically where the encounter takes place with respect to Titan, is dominant. The asymptote correction explained in Sec. III is an external one based on operations. For a Titan encounter, the external asymptote provides sufficient accuracy because the effects of asymptote variations from grid point to grid point are negligible. On the other hand, for an encounter of a less massive body near periapsis, the asymptote can change substantially from one grid point to another, and this asymptote change must be solved for by LAMBIC during its processing in addition to the asymptote correction due to operations. Hence, the grid points, which are based only on two-dimensional variations, would expand to five dimensions. The extra three dimensions would deal with the asymptote differences.

Therefore, an augmented or enhanced grid generation could be added to the LAMBIC software to better account for asymptote variation among grid points

$$K_{6\times3} = \begin{bmatrix} A_{3\times3} \\ S_{3\times3} \end{bmatrix} \tag{11}$$

Assume that $K_{6\times3}$ found in Eq. (11) is the K matrix at the approach (or prior) maneuver point, but it is broken into two 3×3 matrices, A and S. To take into account the asymptote variation among the grid points multiply the $K_{6\times3}$ matrix in the following manner to adjust the asymptotes accordingly:



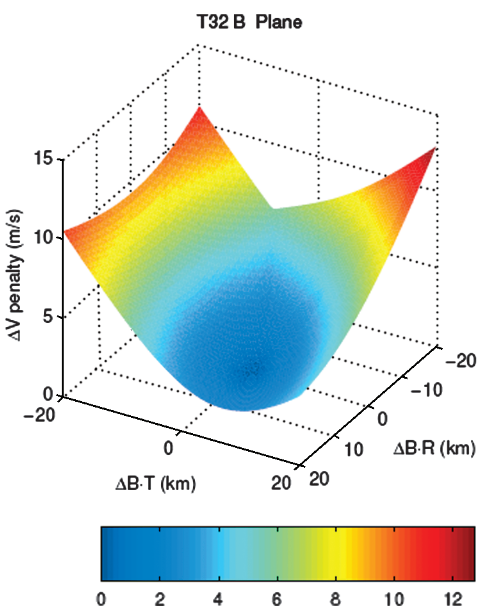


Fig. 9 Contour plot for OTM115 showing the contours with the biased target.

$$K_{6\times3}A_{3\times3}^{-1}\begin{bmatrix}\Delta B\bullet R\\ \Delta B\bullet T\\ 0\end{bmatrix}=\begin{bmatrix}A_{3\times3}\\ S_{3\times3}\end{bmatrix}A_{3\times3}^{-1}\begin{bmatrix}\Delta B\bullet R\\ \Delta B\bullet T\\ 0\end{bmatrix}$$
$$=\begin{bmatrix}I_{3\times3}\\ S_{3\times3}A_{3\times3}^{-1}\end{bmatrix}\begin{bmatrix}\Delta B\bullet R\\ \Delta B\bullet T\\ 0\end{bmatrix} \tag{12}$$

Notice that the $\Delta B \bullet R$ and $\Delta B \bullet T$ remain the same due to the identity matrix (I) and the S matrix contains the desired asymptote adjustments. An adjustment to the LAMBIC software would be the next course of action with some additional work needed to verify the algorithm.

VI. Conclusions

The cleanup maneuver and approach maneuver contour plots used by the Cassini–Huygens maneuver team have helped in the maneuver cancellation decision-making process and have become a valuable tool to save ΔV as the opportunity presents itself. The contour plots started to be used during operations after the Cassini–Huygens maneuver team enhanced the LAMBIC software to account for the change between incoming nominal and perturbed asymptotes. Furthermore, the position variations of an encounter without the time of flight component are a significant source to show the ΔV variations of an encounter. The ability to find a new target quickly and accurately from the contour plots in order to save ΔV is a testament to the usefulness of these plots. Especially due to the short time frames to make decisions and gather data, the contour plots provide a quick analysis that is vital to the Cassini project.

Future missions that involve gravity-assist encounters could use the cost contour plots to produce an efficient mission. It is possible that missions with flexible target requirements and approval from the scientists could save substantial ΔV by implementing a variable target process, essentially allowing the target to change to the target that saves the most ΔV. Understanding the statistics of ΔV cost for different encounter positions would aid other missions in the same manner as Cassini. In addition, there could be substantial savings in ΔV if the cost contour plots are used for interplanetary gravity assists, which would be a new use that was not applicable during the Cassini project.

Appendix: *B* Plane

The B plane in Fig. A1 is another name for the aiming plane, which is a plane that is perpendicular to the incoming asymptote vector of the spacecraft and passes through the center of the target body. The B vector is the vector from the center of the target body to the location in the B plane where the path of the spacecraft is projected onto the B plane. The B vector is also known as the impact parameter and marks the place of closest approach to the target body if the target body was massless. The B vector position can be plotted by using the two principle axes of the B plane: R and T. The T axis is usually aligned with either the ecliptic or the body equatorial plane of the target body. The R axis is the third vector that completes the right-handed system with the S and T axes. The $B \bullet R$ and $B \bullet T$ components provide the position of the encounter in the B plane. The inset on Fig. A1 shows the one-sigma dispersion ellipse that encompasses the impact parameter point. This ellipse has a semimajor axis dimension (SMAA), semiminor axis dimension (SMIA), and an orientation value θ, which is the angle between the T axis and the semimajor axis of the ellipse in the clockwise direction. The S component of the dispersion is a time of flight dispersion or distance dispersion along the S axis.

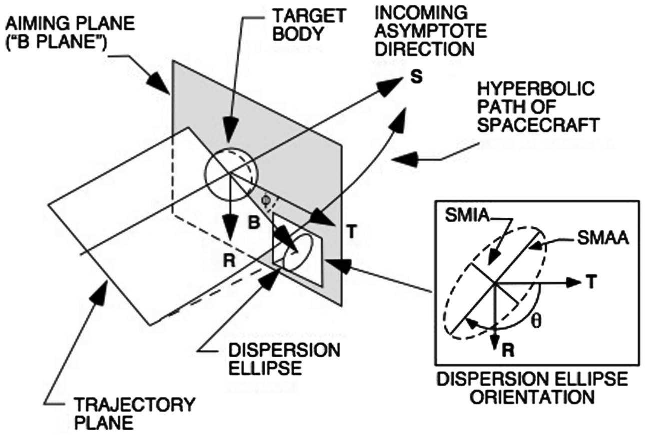


Fig. A1 An example of a *B* plane.

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